Math 6172 Answers to Problems in Test II Spring 2003

1. (a) It is convergent according to the comparison test.
   (b) It is convergent according to the ratio test
   (c)
$$R = \sqrt{2}.$$
   (d)
$$R = \infty.$$

2. (a)
$$\frac{1}{1+z^2} = \sum_{n=0}^{\infty} (-1)^n z^{2n}, \quad \text{for } |z| < 1.$$
   (b)
$$\sin^2 z = \frac{1}{2}\left[\frac{(2z)^2}{2!} - \frac{(2z)^4}{4!} + \cdots + (-1)^{n-1}\frac{(2z)^{2n}}{(2n)!} + \cdots\right], \quad \text{for } |z| < \infty.$$
   (c)
$$\frac{1}{(z+i)^2} = \sum_{n=0}^{\infty} n i^{(n+1)} z^{n-1}, \quad \text{for } |z| < 1.$$
   (d)
$$\int_0^Z \cos t^2 dt = z - \frac{z^5}{5 \cdot 2!} + \frac{z^9}{9 \cdot 4!} + \cdots + (-1)^n \frac{z^{4n+1}}{(4n+1)(2n)!} + \cdots, \quad \text{for } |z| \leq \infty.$$

3.
$$\frac{1}{z+1} = \frac{1}{2}\sum_{n=0}^{\infty}\left(-\frac{z-1}{2}\right)^n, \qquad \text{for } |z-1| < 2.$$
$$\frac{1}{z+1} = \frac{1}{z-1}\sum_{n=0}^{\infty}\left(-\frac{2}{z-1}\right)^n, \qquad \text{for } |z-1| > 2.$$

4.
$$\oint_c \frac{e^z + z}{z^3 - z} dz = -2\pi i.$$

5.

$$\int_0^{2\pi} \frac{1}{2-\cos\theta} d\theta = \frac{2\pi}{\sqrt{3}} \approx 3.6276.$$

6.

$$\int_{-\infty}^{\infty} \frac{\sin 2x}{x^2+x+1} dx = \frac{-2\pi e^{-\sqrt{3}} \sin 1}{\sqrt{3}} \approx -0.54.$$

7.

$$\phi\left(r,\theta\right) = \frac{5}{2} + 2r^2 \cos 2\theta + \frac{3}{2} r^4 \cos 4\theta.$$

8.

$$U = \frac{110}{\pi} \arg\left(-\frac{1+i}{2} \cdot \frac{z-1}{z-i} \right).$$

9.

$$T = \frac{10}{\pi} \arg\left(\frac{z^2-1}{z^2+4} \right).$$