MATH 6172 **Test III** **Spring 2002**

Name : _____

SHOW THE DETAILS OF YOUR WORK ID : _____

1. (a) Find the number of samples of three light bulbs that can be drawn from a box containing 20 distinct bulbs.

   (b) In how many different ways can we select a committee consisting of 2 engineers, 3 chemists and 4 mathematicians from 5 engineers, 6 chemists and 7 mathematicians ?

   (c) If a cage contains 100 mice, 6 of which are male, what is the probability that 4 male mice will be included if 7 mice are randomly selected ?

(d) Suppose that we draw cards repeatedly and with replacement from a file of 100 cards, 20 of which refer to male and 80 to female persons. What is the probability of obtaining the third "female" card before the third "male" card?

(e) (A bonus problem with extra points!!) Suppose that we draw cards repeatedly and with replacement from a file of 100 cards, 20 of which refer to male and 80 to female persons. Show that the probability of obtaining the m-th "female" card before the n-th "male" card is

$$\sum_{x=0}^{n-1} \left(\begin{array}{c} n+m-1 \\ x \end{array} \right) 0.2^x (0.8)^{n+m-1-x}.$$

2. (a) Find the mean of the binomial distribution:

$$f(x) = \left(\begin{array}{c} n \\ x \end{array} \right) p^x (1-p)^{n-x}.$$

(b) Show that the hypergeometric distribution $f(x) = \dfrac{\left(\begin{array}{c} M \\ x \end{array} \right) \left(\begin{array}{c} N-M \\ n-x \end{array} \right)}{\left(\begin{array}{c} N \\ n \end{array} \right)}$

has $E(x) = \dfrac{nM}{N}$.

3. An electronic device consists of two components. Let X and Y [years] be the times to failure of the first and second components, respectively. Assume that (X, Y) has the density $f(x, y) = 9e^{-3(x+y)}$ if $x > 0$ and $y > 0$ and 0 otherwise.

   (a) Are X and Y dependent or independent?

   (b) Find the densities of marginal distributions.

   (c) What is the probability that the first component will have a lifetime of 2 years or longer?

4. Assume that X has a binomial distribution, i.e., the probability of an event A happening x times during n trials is $f(x) = \begin{pmatrix} n \\ x \end{pmatrix} p^x(1-p)^{n-x}$. Suppose that in the first n trails the event A happened x_1 times, in the second n trails A happened x_2 times, $\cdots$, in the k-th n trails A happened x_k times. Find the maximum likelihood estimate for p from the data.

5. (a) Determine a 99% confidence interval for the mean μ of a normal population, using a sample of size 16 with mean $\bar{x} = 30.5$ and sample variance $s^2 = 0.25$.

(b) Using a sample of size 15 with $s^2 = 1.56$ and assuming normality, find a 95% confidence interval for σ^2.

6. (a) Assuming normality and know variance $\sigma^2 = 400$, test the hypothesis $\mu = 40$ against the alternative $\mu = 35$, using a sample of size 100 with mean $\bar{x} = 36$ and choosing the significance level 5%.

(b) Choosing the significance level 1%, do the same problem above.

7. Determine the sample regression line of y on x based on the data:

x	30	40	50	60
y	160	240	330	435

and find the value of y for $x = 35$ from the line.