Math 6172 Answers to Problems in Test III Spring 2002

1. (a)

$$\frac{20 \cdot 19 \cdot 18}{3!} = 1140.$$

(b)

$$\frac{5 \cdot 4}{2!} \cdot \frac{6 \cdot 5 \cdot 4}{3!} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4}{4!} = 7000.$$

(c)

$$\frac{\binom{6}{4}\binom{94}{3}}{\binom{100}{7}} \approx 0.0001256.$$

(d)

$$0.8^5 + 5 \cdot 0.2 \cdot 0.8^4 + 10 \cdot 0.2^2 \cdot 0.8^3 \approx 0.94208$$

or

$$0.8^3 + 3 \cdot 0.2 \cdot 0.8^3 + 6 \cdot 0.2^2 \cdot 0.8^3 = 0.94208.$$

(e) We take $m+n-1$ cards. If the number of male cards is $0, 1, \cdots, n-1$, the number of female cards is greater than or equal to m. Therefore the m-th female card must be before the n-th male card. If the number of male cards is $n, \cdots, m+n-1$, then the number of female cards must be less than or equal to $m-1$. Therefore the m-th female card must be after the n-th male card. The probability of there being x male cards among $m+n-1$ cards is

$$\binom{m+n-1}{x} 0.2^x (0.8)^{m+n-1-x}.$$

Therefore the probability we need is $\sum_{x=0}^{n-1} \binom{m+n-1}{x} 0.2^X (0.8)^{m+n-1-x}$.

2. (a)

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} = np.$$

(b)

$$
\begin{aligned}
& E(x) \\
= & \sum_{x=0}^{n} \frac{x\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} = \sum_{x=1}^{n} \frac{x\dfrac{M!}{x!\,(M-x)!}\cdot\dfrac{(N-M)!}{(n-x)!(N-M-n+x)!}}{\dfrac{N!}{(N-n)!n!}} \\
= & \frac{nM}{N}\sum_{x=1}^{n} \frac{\dfrac{(M-1)!}{(x-1)!\,(m-1-(x-1))!}\cdot\dfrac{(N-1-(M-1))!}{(n-1-(x-1))!\,(N-M-n-1-(x-1))!}}{\dfrac{(N-1)!}{(N-1-(n-1))!\,(n-1)!}} \\
= & \frac{nM}{N}\sum_{x-1=0}^{n-1} \frac{\binom{M-1}{x-1}\binom{N-1-(M-1)}{n-1-(x-1)}}{\binom{N-1}{n-1}} = \frac{nM}{N}.
\end{aligned}
$$

3. (a) Since $f(x,y) = 9e^{-3(x+y)} = 9e^{-3x}\cdot e^{-3y}$, they are independent.

(b)

$$
\begin{aligned}
f_1(x) &= 3e^{-3x}, \\
f_2(y) &= 3e^{-3y}.
\end{aligned}
$$

(c)

$$
P(X \geq 2) = \int_{2}^{\infty} 3e^{-3x}dx = e^{-6} \approx 0.002478752.
$$

4.

$$
p = \frac{\sum_{i=1}^{k} x_i}{nk}.
$$

5. (a)

$$
CONF_{0.99}\left(30.13125 \leq \mu \leq 30.86875\right).
$$

(b)

$$
CONF_{0.95}\left(0.836 \leq \sigma^2 \leq 3.879\right).
$$

6. (a)

$$
P(X \leq c) = 0.05 \quad \Rightarrow \quad c = -1.645
$$

$$
\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{36-40}{\frac{20}{10}} = -2 < 1.645. \qquad \text{Reject.}
$$

(b)

$$P\left(X \leq c\right) = 0.01 \quad \Rightarrow \quad c = -2.326 < -2. \quad \text{Accept.}$$

7.

$$y = a + bx,$$

where

$$\begin{aligned} a &= \frac{\begin{vmatrix} 1165 & 180 \\ 57000 & 8600 \end{vmatrix}}{\begin{vmatrix} 4 & 180 \\ 180 & 8600 \end{vmatrix}} = \frac{-2410}{20} = -120.5, \\ b &= \frac{\begin{vmatrix} 4 & 1165 \\ 1.8 & 570 \end{vmatrix}}{20} = \frac{193}{20} = 9.15. \end{aligned}$$

$$y\left(35\right) = -120.5 + 35 \cdot 9.15 = 199.75.$$