MATH 6172 **Test III** **Spring 2003**

Name : ____________________

SHOW THE DETAILS OF YOUR WORK ID : ____________________

1. (a) In how many ways can a committee of three be chosen from 50 person?

   (b) In how many different ways can seven people be seated at a round table?

   (c) Suppose that a box contains four red and six green balls and that five balls are randomly selected from the box. Find the probability of containing three red balls.

(d) Given the relation $P(A \cap B) = P(B|A)P(A)$, show that $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$.

(e) Suppose that we draw cards repeatedly and with replacement from a file of 100 cards, 10 of which refer to male and 90 to female persons. What is the probability of obtaining the third "female" card before the second "male" card?

(f) (A bonus problem with extra points!!) Suppose that we draw cards repeatedly and with replacement from a file of 100 cards, 20 of which refer to male and 80 to female persons. Show that the probability of obtaining the m-th "female" card before the n-th "male" card is

$$\sum_{x=0}^{n-1} \binom{n+m-1}{x} 0.2^x (0.8)^{n+m-1-x}.$$

2. (a) We know that for the binomial distribution:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x},$$

the mean is equal to $\mathbf{E}[x] = np$. Find its variance.

(b) Show that the normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ has

$$\mathbf{E}[x] = \mu.$$

$$\left(\text{Hint: } \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-y^2/2} dy = 1.\right)$$

3. (a) Show that the two-dimensional random variables with the densities $f(x, y) = x + y$ and $g(x, y) = (x + 1/2)(y + 1/2)$ if $0 \leq x \leq 1$, $0 \leq y \leq 1$, have the same marginal distributions.

(b) If certain sheets of paper have a mean weight of 1.5 grams each, with a standard deviation of 0.02 gram, what are the mean weight and the standard deviation of a pack of 10,000 sheets.

4. Assume that the numbers of defects per unit length of textile, X, has a Poisson distribution, i.e., the probability of existing x defects per unit length of textile is $f(x) = \dfrac{\mu^x}{x!} e^{-\mu}$, $x = 0, 1, \cdots$. Suppose that in the first unit length of textile there are x_1 defects, in the second unit length of textile there are x_2 defects, $\cdots$, in the k-th unit length of textile there are x_k defects. Find the maximum likelihood estimate for μ from the data.

5. (a) Determine a 99% confidence interval for the mean μ of a normal population with a known $\sigma^2 = 0.64$, using a sample of size 36 with mean $\bar{x} = 50.3$.

(b) Using a sample of size 10 with $s^2 = 0.56$ and assuming normality, find a 95% confidence interval for σ^2.

6. (a) Assuming normality, test the hypothesis $\mu = 40$ against the alternative $\mu = 31$. The sample has a size of 100 with mean $\bar{x} = 34$ and sample variance $s^2 = 900$. The significance level is equal to 5%.

   (b) Choosing the significance level 1%, do the same problem above.

7. Can you claim on a 5% level that a die is fair if 60 trails give $1, \cdots, 6$ with absolute frequencies 10, 13, 9, 11, 9, 8?

8. Determine the sample regression line of y on x based on the data:

x	32	50	100	150	212
y	0.337	0.345	0.365	0.380	0.395

and find the value of y for $x = 66$ from the line.