MATH 6204 **Test II** **Spring 2005**

Name :_____

ID :_____

Show all the details of your work !!

1. Consider the following free-boundary problem that is related to American lookback strike put options:

$$\begin{cases} \dfrac{\partial W}{\partial t} + \dfrac{1}{2}\sigma^2\eta^2\dfrac{\partial^2 W}{\partial \eta^2} + (D_0 - r)\,\eta\dfrac{\partial W}{\partial \eta} - D_0 W = 0, & \\ & 1 \le \eta \le \eta_f(t), \quad 0 \le t \le T, \\ W(\eta, T) = \max(\eta - \beta, 0), & 1 \le \eta \le \eta_f(T), \\ \dfrac{\partial W}{\partial \eta}(1, t) = 0, & 0 \le t \le T, \\ W(\eta_f, t) = \eta_f - \beta, & 0 \le t \le T, \\ \dfrac{\partial W}{\partial \eta}(\eta_f, t) = 1, & 0 \le t \le T, \\ \eta_f(T) = \beta \max(1, D_0/r). & \end{cases}$$

   Convert this problem into a problem whose solution has a continuous derivative everywhere. Here we also require that the problem is defined on $[0,1] \times [0,T]$ and with an initial condition. (Assume $1 < \beta$).

2. Design an exponential scheme to approximate

$$a(\xi)\frac{\partial^2 W}{\partial \xi^2} + b(\xi)\frac{\partial W}{\partial \xi} + c(\xi)W,$$

   where $a(\xi) \ge 0$ and $c(\xi) \le 0$. (Let $\xi_{i-1} < \xi_i < \xi_{i+1}$ and W_{i-1}, W_i, W_{i+1} be the values of $W(\xi)$ at $\xi = \xi_{i-1}, \xi_i, \xi_{i+1}$. Find an approximate expression to $a(\xi_i)\dfrac{\partial^2 W_i}{\partial \xi^2} + b(\xi_i)\dfrac{\partial W_i}{\partial \xi} + c(\xi_i)W_i$ in the form of linear combination of W_{i-1}, W_i and W_{i+1}.)

3. Assume that the market price of the risk in the bond equation has been found. Design an implicit second-order accurate finite-difference

method based on the bond equation to solve the European bond option problem. (Suppose that the option is a six-month option on a three year bond with a coupon rate k.)